Online Appendix
Shrinking Factor Dimension: A Reduced-Rank Approach

This appendix provides the complete results for the robustness checks discussed in the paper. Below, we briefly describe the contents of the appendix tables.

- **Table A1:** Performances of factor models estimated with 48 industry portfolios: Alternative measures
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Table A1 Performances of factor models estimated with 48 industry portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.04	0.05	0.07	0.07	0.27	0.37	0.47	0.42
PCA	0.17	0.18	0.16	0.19	1.45	1.54	1.25	1.42
PLS	0.18	0.11	0.07	0.05	1.50	0.70	0.39	0.28
RRA	0.04	0.04	0.04	0.04	0.28	0.25	0.23	0.24
Panel B: 202 characteristic portfolios								
FF	0.09	0.08	0.07	0.06	0.42	0.36	0.25	0.20
PCA	0.25	0.26	0.23	0.26	1.58	1.66	1.33	1.45
PLS	0.28	0.23	0.19	0.14	1.60	0.94	0.59	0.36
RRA	0.09	0.09	0.10	0.12	0.41	0.35	0.34	0.35
Panel C: All stocks								
FF	0.10	0.10	0.11	0.12	3.54	5.51	5.76	6.25
PCA	0.14	0.15	0.15	0.15	3.92	3.88	4.80	4.97
PLS	0.14	0.12	0.12	0.12	3.63	4.28	3.79	3.82
RRA	0.10	0.10	0.11	0.11	3.69	3.65	4.22	3.41
Panel D: All-but-micro stocks								
FF	0.10	0.11	0.12	0.12	2.72	2.80	3.35	3.43
PCA	0.15	0.16	0.16	0.16	2.90	3.41	3.30	3.42
PLS	0.15	0.13	0.12	0.12	2.96	2.83	3.10	3.27
RRA	0.10	0.10	0.11	0.11	2.68	2.83	3.07	2.74

Table A2 Performances of factor models estimated with 202 characteristic portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 202 characteristic portfolios are used as basis assets to estimate the weights of factor proxies, which include 25 size-B/M portfolios, 17 industry portfolios, 25 operating profitability-investment portfolios, 25 size-variance portfolios, 35 size-net issuance portfolios, 25 size-accruals portfolios, 25 size-beta portfolios, and 25 size-momentum portfolios. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.04	0.05	0.07	0.07	0.27	0.37	0.47	0.42
PCA	0.17	0.18	0.16	0.19	1.45	1.54	1.25	1.42
PLS	0.18	0.06	0.06	0.05	1.46	0.42	0.37	0.31
RRA	0.04	0.06	0.06	0.05	0.31	0.38	0.37	0.33
Panel B: 202 characteristic portfolios								
FF	0.09	0.08	0.07	0.06	0.42	0.36	0.25	0.20
PCA	0.25	0.26	0.23	0.26	1.58	1.66	1.33	1.45
PLS	0.27	0.16	0.10	0.10	1.56	0.61	0.28	0.30
RRA	0.09	0.09	0.07	0.07	0.43	0.33	0.22	0.21
Panel C: All stocks								
FF	0.10	0.10	0.11	0.12	3.54	5.51	5.76	6.25
PCA	0.14	0.15	0.15	0.15	3.92	3.88	4.80	4.97
PLS	0.14	0.12	0.12	0.12	3.58	4.30	4.26	4.78
RRA	0.10	0.10	0.11	0.11	4.24	5.04	5.94	5.45
Panel D: All-but-micro stocks								
FF	0.10	0.11	0.12	0.12	2.72	2.80	3.35	3.43
PCA	0.15	0.16	0.16	0.16	2.90	3.41	3.30	3.42
PLS	0.15	0.12	0.12	0.12	2.87	2.75	3.21	3.29
RRA	0.10	0.11	0.11	0.11	3.03	2.78	3.28	3.21

Table A3 Out-of-sample performance of factor models estimated with 48 industry portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 30-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 13-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 2006:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.09	0.10	0.10	0.08	0.69	0.74	0.81	0.70
PCA	0.19	0.18	0.16	0.16	1.30	1.26	1.04	1.04
PLS	0.18	0.11	0.08	0.08	1.21	0.66	0.62	0.62
RRA	0.09	0.08	0.07	0.07	0.69	0.54	0.50	0.48
Panel B: 202 characteristic portfolios								
FF	0.09	0.11	0.09	0.09	0.42	0.41	0.32	0.29
PCA	0.26	0.25	0.21	0.21	1.41	1.37	1.11	1.14
PLS	0.25	0.16	0.09	0.09	1.30	0.66	0.36	0.34
RRA	0.09	0.08	0.08	0.11	0.41	0.35	0.34	0.32
Panel C: All stocks								
FF	0.11	0.11	0.12	0.11	2.24	2.25	2.20	2.28
PCA	0.15	0.15	0.14	0.14	1.92	2.13	2.07	2.11
PLS	0.15	0.13	0.12	0.12	1.71	2.13	2.41	2.49
RRA	0.11	0.11	0.11	0.12	2.14	2.12	2.49	2.38
Panel D: All-but-micro stocks								
FF	0.12	0.12	0.13	0.12	2.80	2.57	2.72	2.75
PCA	0.17	0.16	0.15	0.15	2.17	2.35	2.32	2.36
PLS	0.17	0.13	0.12	0.12	1.89	2.49	3.05	3.08
RRA	0.12	0.12	0.12	0.12	2.66	2.75	2.82	2.53

Table A4 Out-of-sample performances of factor models estimated with 202 characteristic portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 202 characteristic portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 30-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 13-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 2006:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.09	0.10	0.10	0.08	0.69	0.74	0.81	0.70
PCA	0.19	0.18	0.16	0.16	1.30	1.26	1.04	1.04
PLS	0.18	0.10	0.08	0.08	1.21	0.64	0.69	0.64
RRA	0.09	0.09	0.07	0.07	0.69	0.68	0.51	0.53
Panel B: 202 characteristic portfolios								
FF	0.09	0.11	0.09	0.09	0.42	0.41	0.32	0.29
PCA	0.26	0.25	0.21	0.21	1.41	1.37	1.11	1.14
PLS	0.25	0.13	0.07	0.07	1.31	0.53	0.34	0.33
RRA	0.10	0.10	0.09	0.09	0.41	0.38	0.31	0.31
Panel C: All stocks								
FF	0.11	0.11	0.12	0.11	2.24	2.25	2.20	2.28
PCA	0.15	0.15	0.14	0.14	1.92	2.13	2.07	2.11
PLS	0.15	0.12	0.12	0.11	1.75	2.08	2.49	2.49
RRA	0.11	0.11	0.11	0.11	2.08	2.18	2.28	2.33
Panel D: All-but-micro stocks								
FF	0.12	0.12	0.13	0.12	2.80	2.57	2.72	2.75
PCA	0.17	0.16	0.15	0.15	2.17	2.35	2.32	2.36
PLS	0.17	0.13	0.12	0.12	1.96	2.55	3.20	3.21
RRA	0.12	0.12	0.11	0.11	2.60	2.61	2.72	2.71

Table A5 Out-of-sample performances of factor models estimated with 48 industry portfolios: Alternative periods

This table reports the out-of-sample root-mean-squared alphas and total adj-R^2s of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 20-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 23-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 2006:01–2016:12.

	Root-mean-squared alpha (%)				Total adj-R^2 (%)			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.30	0.33	0.40	0.35	43.91	47.16	44.46	44.23
PCA	0.68	0.92	0.85	0.78	9.30	24.09	22.68	21.82
PLS	0.81	0.57	0.33	0.31	24.32	31.02	43.60	43.53
RRA	0.32	0.31	0.31	0.27	43.93	46.94	48.55	47.27
Panel B: 202 characteristic portfolios								
FF	0.31	0.25	0.23	0.19	68.64	82.30	82.22	84.10
PCA	0.72	1.09	0.93	0.86	19.41	49.42	49.87	49.60
PLS	0.99	0.70	0.31	0.33	49.61	58.77	74.51	74.49
RRA	0.33	0.30	0.31	0.28	72.23	75.31	79.87	79.70
Panel C: All stocks								
FF	3.47	3.43	3.75	3.95	3.99	−0.60	−12.08	−19.28
PCA	3.48	3.77	3.89	3.99	−1.88	−2.35	−14.23	−20.54
PLS	3.62	3.75	3.89	3.89	4.32	−1.74	−10.80	−16.21
RRA	3.44	3.58	3.58	3.63	5.27	−1.26	−9.52	−15.30
Panel D: All-but-micro stocks								
FF	2.45	2.46	2.57	2.58	14.25	11.59	1.79	−4.81
PCA	2.60	2.74	2.80	2.84	−0.12	2.84	−6.95	−11.86
PLS	2.59	2.65	2.67	2.70	9.19	5.62	0.32	−4.02
RRA	2.45	2.51	2.49	2.56	15.43	10.51	4.83	−0.52

Table A6 Out-of-sample performances of factor models estimated with 202 characteristic portfolios: Alternative periods

This table reports the out-of-sample root-mean-squared alphas and total adj-R^2s of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 1, 3, 5, and 6 factor(s) are the market factor, FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. PCA, PLS, and RRA refer to the factors based on the principal component analysis, partial least squares, and reduced-rank approach, respectively. When extracting the PLS and RRA factors, the 202 characteristic portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 20-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 23-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 1996:01–2016:12.

	Root-mean-squared alpha (%)				Total adj-R^2 (%)			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
FF	0.30	0.33	0.40	0.35	43.91	47.16	44.46	44.23
PCA	0.68	0.92	0.85	0.78	9.30	24.09	22.68	21.82
PLS	0.80	0.37	0.33	0.32	25.39	34.76	44.50	43.56
RRA	0.32	0.32	0.28	0.25	42.93	47.86	47.14	45.80
Panel B: 202 characteristic portfolios								
FF	0.31	0.25	0.23	0.19	68.64	82.30	82.22	84.10
PCA	0.72	1.09	0.93	0.86	19.41	49.42	49.87	49.60
PLS	0.97	0.41	0.32	0.35	52.24	63.91	75.74	76.14
RRA	0.32	0.29	0.21	0.20	75.55	82.16	83.79	83.50
Panel C: All stocks								
FF	3.47	3.43	3.75	3.95	3.99	−0.60	−12.08	−19.28
PCA	3.48	3.77	3.89	3.99	−1.88	−2.35	−14.23	−20.54
PLS	3.60	3.77	3.94	3.96	4.83	−1.86	−10.68	−16.45
RRA	3.40	3.49	3.87	3.88	6.20	−1.36	−11.79	−17.20
Panel D: All-but-micro stocks								
FF	2.45	2.46	2.57	2.58	14.25	11.59	1.79	−4.81
PCA	2.60	2.74	2.80	2.84	−0.12	2.84	−6.95	−11.86
PLS	2.57	2.62	2.66	2.67	9.84	6.45	0.52	−4.22
RRA	2.49	2.45	2.56	2.61	16.04	11.22	2.12	−2.66

Table A7 Performance of RRA factors given pricing error restrictions and estimated with 48 industry portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of RRA factors in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. The RRA factors are extracted by using the reduced-rank approach and are assumed to have pricing error as $\alpha_i = \eta\sigma_i$, where σ_i is asset i's volatility. When extracting the RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
$\eta=0$	0.04	0.03	0.03	0.04	0.28	0.23	0.20	0.20
$\eta=0.5/12$	0.06	0.06	0.06	0.06	0.35	0.31	0.28	0.26
$\eta=1/12$	0.10	0.10	0.10	0.10	0.54	0.52	0.49	0.49
$\eta=1.5/12$	0.14	0.15	0.15	0.15	0.80	0.79	0.77	0.76
Panel B: 202 characteristic portfolios								
$\eta=0$	0.09	0.10	0.11	0.13	0.41	0.36	0.36	0.36
$\eta=0.5/12$	0.14	0.15	0.16	0.18	0.47	0.44	0.42	0.44
$\eta=1/12$	0.19	0.21	0.22	0.24	0.61	0.61	0.60	0.63
$\eta=1.5/12$	0.25	0.27	0.29	0.30	0.82	0.84	0.84	0.87
Panel C: All stocks								
$\eta=0$	0.10	0.10	0.11	0.11	3.70	3.34	3.77	3.32
$\eta=0.5/12$	0.10	0.11	0.11	0.12	3.13	2.89	3.27	2.91
$\eta=1/12$	0.11	0.12	0.12	0.12	2.66	2.56	2.89	2.59
$\eta=1.5/12$	0.12	0.12	0.13	0.13	2.27	2.46	2.88	2.83
Panel D: All-but-micro stocks								
$\eta=0$	0.10	0.10	0.11	0.11	2.70	2.84	3.01	2.82
$\eta=0.5/12$	0.10	0.11	0.11	0.12	2.22	2.54	2.70	2.53
$\eta=1/12$	0.11	0.12	0.12	0.12	1.83	2.33	2.49	2.43
$\eta=1.5/12$	0.12	0.13	0.13	0.14	1.73	2.30	2.47	2.44

Table A8 Performance of RRA factors given pricing error restrictions and estimated with 202 characteristic portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of RRA factors in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. The RRA factors are extracted by using the reduced-rank approach and are assumed to have pricing error as $\alpha_i = \eta\sigma_i$, where σ_i is asset i's volatility. When extracting the RRA factors, the 202 characteristic portfolios are used as basis assets to estimate the weights of factor proxies. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
$\eta=0$	0.04	0.07	0.06	0.05	0.32	0.44	0.38	0.33
$\eta=0.5/12$	0.05	0.06	0.05	0.05	0.31	0.37	0.32	0.28
$\eta=1/12$	0.07	0.07	0.07	0.07	0.42	0.36	0.36	0.35
$\eta=1.5/12$	0.10	0.09	0.10	0.10	0.57	0.47	0.48	0.50
Panel B: 202 characteristic portfolios								
$\eta=0$	0.09	0.08	0.06	0.06	0.43	0.30	0.21	0.20
$\eta=0.5/12$	0.11	0.11	0.10	0.10	0.43	0.31	0.25	0.24
$\eta=1/12$	0.16	0.17	0.17	0.17	0.51	0.42	0.40	0.40
$\eta=1.5/12$	0.21	0.24	0.24	0.24	0.63	0.60	0.61	0.60
Panel C: All stocks								
$\eta=0$	0.10	0.11	0.11	0.11	4.38	5.36	6.26	5.79
$\eta=0.5/12$	0.10	0.11	0.11	0.12	3.75	4.93	5.91	5.37
$\eta=1/12$	0.10	0.11	0.12	0.12	3.09	4.49	5.67	5.00
$\eta=1.5/12$	0.11	0.12	0.12	0.12	2.55	4.10	5.47	4.82
Panel D: All-but-micro stocks								
$\eta=0$	0.10	0.11	0.11	0.12	3.16	2.87	3.52	3.29
$\eta=0.5/12$	0.10	0.11	0.12	0.12	2.64	2.63	3.34	3.11
$\eta=1/12$	0.11	0.11	0.12	0.12	2.15	2.40	3.22	3.00
$\eta=1.5/12$	0.11	0.12	0.13	0.13	1.85	2.33	3.12	2.94

Table A9 Out-of-sample performance of RRA factors given pricing error restrictions and estimated with 48 industry portfolios: Alternative measures

This table reports the out-of-sample volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of RRA factors in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. The RRA factors are extracted by using the reduced-rank approach and are assumed to have pricing error as $\alpha_i = \eta\sigma_i$, where σ_i is asset i's volatility. When extracting the RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 30-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 13-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 2006:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
$\eta=0$	0.09	0.08	0.07	0.07	0.70	0.55	0.49	0.49
$\eta=0.5/12$	0.09	0.08	0.07	0.07	0.68	0.56	0.48	0.44
$\eta=1/12$	0.09	0.08	0.07	0.08	0.66	0.55	0.46	0.43
$\eta=1.5/12$	0.09	0.07	0.07	0.10	0.64	0.52	0.46	0.46
Panel B: 202 characteristic portfolios								
$\eta=0$	0.09	0.08	0.08	0.11	0.42	0.34	0.33	0.33
$\eta=0.5/12$	0.09	0.08	0.08	0.13	0.41	0.34	0.32	0.37
$\eta=1/12$	0.09	0.07	0.08	0.15	0.40	0.34	0.32	0.44
$\eta=1.5/12$	0.08	0.07	0.08	0.17	0.40	0.34	0.35	0.55
Panel C: All stocks								
$\eta=0$	0.11	0.11	0.11	0.11	2.16	2.12	2.42	2.22
$\eta=0.5/12$	0.11	0.11	0.11	0.12	2.11	2.06	2.38	2.20
$\eta=1/12$	0.11	0.11	0.11	0.12	2.02	1.98	2.33	2.20
$\eta=1.5/12$	0.12	0.11	0.11	0.12	1.92	1.86	2.30	2.23
Panel D: All-but-micro stocks								
$\eta=0$	0.12	0.12	0.11	0.12	2.69	2.69	2.74	2.37
$\eta=0.5/12$	0.12	0.12	0.11	0.12	2.63	2.65	2.67	2.28
$\eta=1/12$	0.12	0.12	0.12	0.12	2.52	2.53	2.57	2.24
$\eta=1.5/12$	0.12	0.12	0.12	0.13	2.41	2.34	2.45	2.25

Table A10 Out-of-sample performance of RRA factors given pricing error restrictions and estimated with 202 characteristic portfolios: Alternative measures

This table reports the out-of-sample volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of RRA factors in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. The RRA factors are extracted by using the reduced-rank approach and are assume to have pricing error as $\alpha_i = \eta\sigma_i$, where σ_i is asset i's volatility. When extracting the RRA factors, the 48 industry portfolios are used as basis assets to estimate the weights of factor proxies. We use the first 30-year data to estimate the weights of the PCA, PLS, and RRA factors and the rest 13-year data for out-of-sample evaluation. For each individual asset, we employ a 60-month rolling window approach, requiring at least 24 observations, to calculate the out-of-sample root-mean-squared alphas and total adj-R^2s. As such, the out-of-sample evaluation period is 2006:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	1	3	5	6	1	3	5	6
Panel A: 48 industry portfolios								
$\eta=0$	0.09	0.09	0.07	0.07	0.69	0.68	0.55	0.53
$\eta=0.5/12$	0.09	0.09	0.07	0.07	0.68	0.67	0.55	0.53
$\eta=1/12$	0.09	0.08	0.07	0.07	0.67	0.66	0.54	0.53
$\eta=1.5/12$	0.09	0.08	0.07	0.07	0.65	0.64	0.53	0.54
Panel B: 202 characteristic portfolios								
$\eta=0$	0.10	0.10	0.09	0.09	0.42	0.37	0.31	0.31
$\eta=0.5/12$	0.10	0.10	0.09	0.09	0.41	0.37	0.31	0.31
$\eta=1/12$	0.10	0.09	0.08	0.08	0.40	0.37	0.30	0.31
$\eta=1.5/12$	0.09	0.09	0.07	0.08	0.41	0.37	0.30	0.31
Panel C: All stocks								
$\eta=0$	0.11	0.11	0.11	0.11	2.11	2.19	2.28	2.45
$\eta=0.5/12$	0.11	0.11	0.11	0.11	2.08	2.19	2.31	2.44
$\eta=1/12$	0.12	0.12	0.11	0.11	2.02	2.20	2.34	2.42
$\eta=1.5/12$	0.12	0.12	0.11	0.11	1.95	2.21	2.39	2.44
Panel D: All-but-micro stocks								
$\eta=0$	0.12	0.12	0.12	0.12	2.63	2.62	2.70	2.76
$\eta=0.5/12$	0.12	0.12	0.12	0.12	2.58	2.60	2.68	2.76
$\eta=1/12$	0.12	0.12	0.12	0.12	2.51	2.55	2.66	2.75
$\eta=1.5/12$	0.12	0.12	0.11	0.12	2.42	2.50	2.64	2.73

Table A11 Performance of factor models with pre-specified FF five factors: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF5 refers to the FF five-factor model, and FF5+PCA, FF5+PLS, and FF5+RRA refer to K-factor models that include FF5 and $(K-5)$ PCA, PLS, and RRA factors, respectively. PCA factors are extracted from the factor proxies excluding the FF five factors; PLS(R48) and RRA(R48) refer to factors estimated with the 48 industry portfolios as basis assets, and etc. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$					$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$				
$K=$	5	6	7	8	10	5	6	7	8	10
Panel A: 48 industry portfolios										
FF5	0.07	–	–	–	–	0.47	–	–	–	–
FF5+PCA	–	0.07	0.07	0.07	0.06–		0.42	0.42	0.43	0.36
FF5+PLS(R48)	–	0.08	0.07	0.06	0.05–		0.46	0.40	0.36	0.29
FF5+PLS(R202)	–	0.07	0.06	0.06	0.05–		0.40	0.36	0.34	0.30
FF5+RRA(R48)	–	0.08	0.07	0.05	0.05–		0.47	0.40	0.28	0.26
FF5+RRA(R202)	–	0.06	0.07	0.06	0.06–		0.38	0.41	0.39	0.37
Panel B: 202 characteristic portfolios										
FF5	0.07	–	–	–	–	0.25	–	–	–	–
FF5+PCA	–	0.06	0.06	0.06	0.06	–	0.19	0.19	0.19	0.18
FF5+PLS(R48)	–	0.07	0.06	0.06	0.06	–	0.24	0.19	0.18	0.17
FF5+PLS(R202)	–	0.06	0.06	0.06	0.07	–	0.18	0.19	0.19	0.18
FF5+RRA(R48)	–	0.07	0.06	0.07	0.07	–	0.25	0.21	0.20	0.19
FF5+RRA(R202)	–	0.06	0.06	0.07	0.07	–	0.19	0.19	0.20	0.18
Panel C: All stocks										
FF5	0.11	–	–	–	–	5.76	–	–	–	–
FF5+PCA	–	0.12	0.12	0.13	0.13	–	6.07	5.79	6.57	7.56
FF5+PLS(R48)	–	0.12	0.12	0.13	0.14	–	6.08	6.33	6.52	6.64
FF5+PLS(R202)	–	0.12	0.12	0.13	0.13	–	6.70	6.29	6.60	6.88
FF5+RRA(R48)	–	0.12	0.12	0.12	0.13	–	5.60	6.14	5.90	5.82
FF5+RRA(R202)	–	0.12	0.12	0.12	0.13	–	6.15	5.74	5.33	6.30
Panel D: All-but-micro stocks										
FF5	0.12	–	–	–	–	3.35	–	–	–	–
FF5+PCA	–	0.12	0.13	0.13	0.14	–	3.84	3.86	3.74	4.06
FF5+PLS(R48)	–	0.12	0.12	0.13	0.14	–	3.59	3.79	3.81	4.02
FF5+PLS(R202)	–	0.12	0.13	0.13	0.14	–	3.71	3.84	3.93	3.92
FF5+RRA(R48)	–	0.12	0.12	0.13	0.14	–	3.60	3.54	3.34	3.48
FF5+RRA(R202)	–	0.12	0.12	0.13	0.13	–	3.32	3.66	3.55	3.52

Table A12 Performance of DMRS factors

This table reports the in- and out-of-sample performances of Daniel, Mota, Rottke, and Santos (DMRS, 2018) model in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. To calculate the out-of-sample performance, we employ a 60-month rolling window approach, requiring at least 24 observations, and focus on the 2006:01–2016:12 period as other tables.

	$\sqrt{\frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^2}$			Total adj-R^2			$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$			$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$		
$K=$	1	3	5	1	3	5	1	3	5	1	3	5
Panel A: In-sample performance												
48 industries	0.26	0.24	0.30	21.07	23.50	24.24	0.04	0.03	0.04	0.31	0.32	0.36
202 portfolios	0.42	0.23	0.30	24.71	33.62	35.33	0.07	0.03	0.06	0.45	0.33	0.37
All stocks	2.80	3.06	3.47	3.44	5.37	6.46	0.11	0.11	0.12	3.30	5.94	6.67
All-but-micro	1.89	2.09	2.43	7.75	10.14	11.58	0.10	0.11	0.13	2.12	2.89	3.53
Panel B: Out-of-sample performance												
48 industries	0.34	0.37	0.40	14.65	11.35	6.32	0.05	0.05	0.05	0.52	0.49	0.53
202 portfolios	0.25	0.28	0.29	21.08	20.93	15.73	0.04	0.05	0.05	0.35	0.34	0.34
All stocks	2.92	2.97	3.06	−2.60	−9.96	−19.78	0.11	0.11	0.12	2.00	2.16	2.45
All-but-micro	2.03	2.06	2.08	1.16	−5.75	−15.15	0.11	0.11	0.11	2.45	2.16	2.41

Table A13 Performance of factor models incorporating Lettau and Pelger (2018) five factors and estimated with 48 industry portfolios

This table reports the root-mean-squared alphas and total adj-R^2s of different K-factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 3, 5, and 6 factor(s) are the FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. RP-PCA refer to factors extracted from the 48 industry portfolios with Lettau and Pelger's (2018) approach. PLS and RRA refer to factors extracted from the combined set of five RP-PCA factors and our factor proxies with the PLS and RRA, where the factors weights are estimated with 48 industry portfolios as basis assets. The sample period is 1974:01–2016:12.

	Root-mean-squared alpha (%)				Total adj-R^2 (%)			
$K =$	3	5	6	10	3	5	6	10
Panel A: 48 industry portfolios								
FF	0.29	0.34	0.31	–	55.57	57.77	58.34	–
RP-PCA	0.18	0.15	0.15	0.13	67.02	73.74	75.67	81.32
PLS	0.23	0.21	0.21	0.17	65.76	71.91	73.02	75.53
RRA	0.21	0.15	0.15	0.15	64.62	73.53	74.58	77.42
Panel B: 202 characteristic portfolios								
FF	0.26	0.19	0.15	–	85.60	86.94	88.30	–
RP-PCA	0.28	0.31	0.31	0.33	78.18	81.01	81.16	83.85
PLS	0.36	0.37	0.36	0.21	81.45	83.70	84.18	87.31
RRA	0.30	0.31	0.30	0.32	79.94	81.37	83.04	86.46
Panel C: All stocks								
FF	2.61	3.08	3.19	–	13.64	14.70	15.55	–
RP-PCA	2.61	2.66	2.73	3.04	11.63	13.54	13.86	15.26
PLS	2.77	2.87	2.89	3.31	13.30	14.62	15.00	16.20
RRA	2.54	2.64	2.68	3.02	12.56	13.70	14.19	16.01
Panel D: All-but-micro stocks								
FF	1.85	2.06	2.09	–	26.98	28.39	29.25	–
RP-PCA	1.76	1.82	1.82	1.87	25.34	28.75	29.27	31.78
PLS	1.92	1.93	1.91	2.06	27.66	29.46	30.24	32.26
RRA	1.79	1.80	1.77	1.94	27.45	29.01	29.89	32.48

Table A14 Performance of factor models incorporating Lettau and Pelger (2018) five factors and estimated with 48 industry portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different K-factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 3, 5, and 6 factor(s) are the FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. RP-PCA refer to factors extracted from the 48 industry portfolios with Lettau and Pelger's (2018) approach. PLS and RRA refer to factors extracted from the combined set of five RP-PCA factors and our factor proxies with the PLS and RRA, where the factors weights are estimated with 48 industry portfolios as basis assets. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	3	5	6	10	3	5	6	10
Panel A: 48 industry portfolios								
FF	0.05	0.07	0.07	–	0.37	0.47	0.42	–
RP-PCA	0.06	0.05	0.04	0.04	0.39	0.27	0.22	0.20
PLS	0.05	0.05	0.05	0.04	0.28	0.26	0.26	0.20
RRA	0.04	0.03	0.03	0.04	0.24	0.18	0.18	0.18
Panel B: 202 characteristic portfolios								
FF	0.08	0.07	0.06	–	0.36	0.25	0.20	–
RP-PCA	0.11	0.08	0.08	0.07	0.34	0.24	0.25	0.21
PLS	0.12	0.13	0.13	0.09	0.36	0.36	0.36	0.22
RRA	0.10	0.11	0.11	0.12	0.38	0.38	0.37	0.34
Panel C: All stocks								
FF	0.10	0.11	0.12	–	5.51	5.76	6.25	–
RP-PCA	0.11	0.12	0.12	0.13	4.62	5.48	5.74	6.33
PLS	0.11	0.11	0.12	0.13	4.25	4.61	4.45	4.57
RRA	0.10	0.11	0.11	0.13	3.57	4.24	4.37	5.14
Panel D: All-but-micro stocks								
FF	0.11	0.12	0.12	–	2.80	3.35	3.43	–
RP-PCA	0.11	0.12	0.12	0.13	2.91	3.43	3.41	3.15
PLS	0.11	0.11	0.11	0.13	2.38	2.59	2.55	2.96
RRA	0.10	0.11	0.11	0.13	2.66	2.60	2.56	2.44

Table A15 Performance of factor models incorporating Lettau and Pelger (2018) five factors and estimated with 202 characteristic portfolios: Alternative measures

This table reports the volatility- and mean-adjusted absolute alphas (i.e., $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\sigma}_{\alpha_i}|$ and $\frac{1}{N}\sum_{i=1}^{N}|\hat{\alpha}_i/\hat{\mu}_i|$) of different K-factor models in explaining four sets of test assets: 48 industry portfolios, 202 characteristic portfolios (Giglio and Xiu, 2019), all stocks, and all-but-micro stocks, respectively. FF refers to the Fama-French model, where 3, 5, and 6 factor(s) are the FF (1993) three factors, FF (2015) five factors, and FF five factors plus the momentum factor, respectively. RP-PCA refer to factors extracted from the 202 characteristic portfolios with Lettau and Pelger's (2018) approach. PLS and RRA refer to factors extracted from the combined set of five RP-PCA factors and our factor proxies with the PLS and RRA, where the factors weights are estimated with 202 characteristic portfolios as basis assets. The sample period is 1974:01–2016:12.

	$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\sigma}_{\alpha_i}}\right\rvert$				$\frac{1}{N}\sum_{i=1}^{N}\left\lvert\frac{\hat{\alpha}_i}{\hat{\mu}_i}\right\rvert$			
$K=$	3	5	6	10	3	5	6	10
Panel A: 48 industry portfolios								
FF	0.05	0.07	0.07	–	0.37	0.47	0.42	–
PCA	0.09	0.07	0.06	0.05	0.54	0.41	0.36	0.31
PLS	0.07	0.07	0.06	0.06	0.45	0.41	0.38	0.32
RRA	0.07	0.07	0.06	0.06	0.44	0.40	0.38	0.35
Panel B: 202 characteristic portfolios								
FF	0.08	0.07	0.06	–	0.36	0.25	0.20	–
PCA	0.08	0.07	0.07	0.07	0.25	0.21	0.21	0.18
PLS	0.09	0.07	0.07	0.07	0.29	0.20	0.20	0.18
RRA	0.08	0.07	0.07	0.07	0.32	0.21	0.21	0.19
Panel C: All stocks								
FF	0.10	0.11	0.12	–	5.51	5.76	6.25	–
PCA	0.11	0.12	0.12	0.14	6.21	7.04	6.91	6.68
PLS	0.11	0.11	0.12	0.13	4.92	6.78	6.77	6.89
RRA	0.10	0.11	0.11	0.14	5.25	6.58	7.47	8.65
Panel D: All-but-micro stocks								
FF	0.11	0.12	0.12	–	2.80	3.35	3.43	–
PCA	0.12	0.12	0.13	0.14	3.35	3.94	3.55	3.52
PLS	0.11	0.12	0.12	0.14	2.82	3.72	3.56	3.71
RRA	0.11	0.12	0.12	0.14	2.98	3.40	3.39	3.50

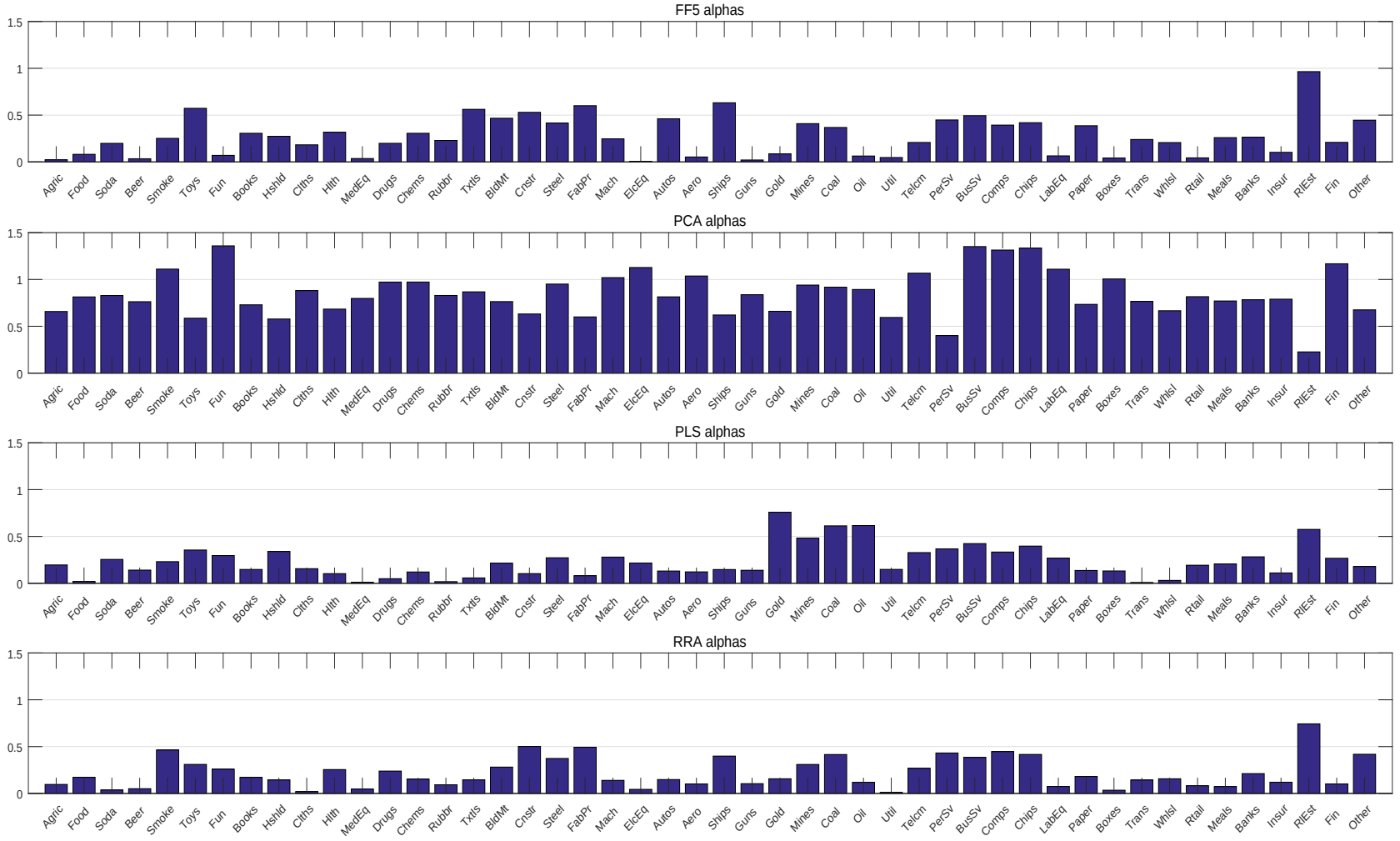

Figure A1 Absolute alphas of 48 industry portfolios (basis assets: 202 characteristic portfolios)

This figure plots the absolute alphas of 48 industry portfolios with the FF, PCA, PLS, and RRA five-factor models, respectively. The PLS and RRA factors are estimated with the 202 characteristic portfolios as basis assets. The sample period is 1974:01–2016:12.

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